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ORIGINAL RESEARCH

A comparative study of two models SV with MCMC algorithm

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Abstract This paper examines two asymmetric stochastic volatility models used to describe the volatility dependencies found in most financial returns. The first is the autoregressive stochastic volatility model with Student's t-distribution (ARSV-t), and the second is the basic SVOL of Jacquier et al. (J Bus Econ Stat 14:429–434, 1994). In order to estimate these models, our analysis is based on the Markov Chain Monte-Carlo (MCMC) method. Therefore, the technique used is a Metropolishastings (Hastings in Biometrika 57:97–109, 1970), and the Gibbs sampler (Casella and George in The Am Stat 46:167–174, 1992; Gelfand and smith in J Am Stat Assoc 85:398–409, 1990; Gilks and Wild in 41:337–348, 1992). The empirical results concerned on the Standard and Poor's 500 composite Index (S&P), CAC40, Nasdaq, Nikkei and DowJones stock price indexes reveal that the ARSV-t model provides a better performance than the SVOL model on the MSE and the maximum Likelihood function.

Keywords Autoregression · MCMC · Stochastic volatility · Student's t-distribution · SVOL

JEL Classification C11 · C15 · C22 · G12

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1 Introduction

Stochastic volatility (SV) models are workhorses for the modelling and prediction of time-varying volatility on financial markets and are essential tools in risk management, asset pricing and asset allocation. In financial mathematics and financial economics, stochastic volatility is typically modelled in a continuous time setting which is advantageous for derivative pricing and portfolio optimization. Nevertheless, since data is typically only observable at discrete points in time, in empirical applications, discrete-time formulations of SV models are equally important.

Volatility plays an important role in determining the overall risk of a portfolio and identifying hedging strategies that make the portfolio neutral with respect to market moves. Moreover, volatility forecasting is also crucial in derivatives trading.

Recently, SV models allowing the mean level of volatility to 'jump' have been used in the literature, see Chang et al. (2007); Chib et al. (2002) and Chib et al. (2002). The volatility of financial markets is a subject of constant analysis movements in the price of financial assets which directly affects the wealth of individual, companies, charities, and other corporate bodies. Determining whether there are any patterns in the size and frequency of such movements, or in their cause and effect, is critical in devising strategies for investments at the micro level and monetary stability at the macro level. Shephard and Pitt (1997) used improved and efficient Markov Chain Monte-Carlo (MCMC) methods to estimate the volatility process "in block" rather than one point of time such as highlighted by Jacquier et al. (1994), for a simple SV model. Furthermore, Hsu and Chiao (2010) analyse the time patterns of individual analyst's relative accuracy ranking in earnings forecasts using a Markov Chain model by treating two levels of stochastic persistence.

Least squares and maximum likelihood techniques have long been used in parameter estimation problems.

However, those techniques provide only point estimates with unknown or approximate uncertainty information. Bayesian inference coupled with the Gibbs Sampler is an approach to parameter estimation that exploits modern computing technology. The estimation results are complete with exact uncertainty information. Section 2 presents the Bayesian approach and the MCMC algorithms. The SV model is introduced in Sect. 3, whereas empirical illustrations are given in Sect. 4.

2 The Bayesian approach and the MCMC algorithm

The Bayesian approach is a classical methodology where we assume that there is a set of unknown parameters. Alternatively, in the Bayesian approach the parameters are considered as random-variables with a given priors distributions. We then use observations (the likelihood) to update these distributions and obtain the posterior distributions.

Formally, let X = ($X_1,\ldots,X_T$) denote the observed data, and θ a parameter vector.

$$P(\theta/X) \propto P(X/\theta) * P(\theta).$$

The posterior distribution $P(\theta/X)$ of a parameter θ/given the observed data X, where $P\ (X/\theta)$ denotes the likelihood distribution of X and $P(\theta)$ denotes the prior distribution of θ.

It would seem that in order to be as subjective as possible and to use the observations as much as possible, one should use priors that are non informative. However this can sometimes create degeneracy issues and one should choose a different prior for this reason.

Markov Chain Monte-Carlo (MCMC) includes the Gibbs Sampler as well as the Metropolis -Hastings (MH) algorithm.

2.1 The metropolis hasting

The Metropolis-Hasting is the baseline for MCMC schemes that simulate a Markov chain $\theta^{(t)}$ with $P(\theta/Y)$ as the stationary distribution of a parameter θ given a stock price index X. For example we can define θ_1, θ_2 and θ_3 such that $\theta = (\theta_1, \theta_2, \theta_3)$ where each θ_i can be scalar, vectors or matrices. Markov Chain Monte-Carlo algorithms are iterative and so at iteration t we will sample in turn from the three conditional distributions. Firstly, we update θ_1 by drawing a value $\theta_1^{(t)}$ from $p(\theta_1/Y,\ \theta_2^{(t-1)},\ \theta_3^{(t-1)})$. Secondly we draw a value for $\theta_2^{(t)}$ from $p(\theta_2/Y,\ \theta_1^{(t)},\ \theta_3^{(t-1)})$ and finally we draw $\theta_3^{(t)}$ from $p(\theta_3/Y,\ \theta_1^{(t)},\ \theta_2^{(t)})$.

We start the algorithm off by selecting initial values, $\theta_i^{(0)}$, for the three parameters. Then sampling from the three conditional distributions in turn will produce a set of Markov chains whose equilibrium distributions can be show to be the joint posterior distributions that we require.

Following Hastings (1970), a generic step from a MH algorithm to update parameter θ_i at iteration t as follows:

1. Sample θ_i^* from the proposal distribution $p_t(\theta_i/\theta_i^{(t-1)})$.
2. Calculate $f = p_t(\theta_i^{(t-1)}/\theta_i^*)/p_t(\theta_i^*/\theta_i^{(t-1)})$ which is known as the Hasting ratio and which equal 1 for symmetric proposals as used in pure Metropolis sampling.
3. Calculate $s_t = fp(\theta_i^*/Y,\phi_i)/p(\theta_i^{(t-1)}/Y,\phi_i)$,where $_i$ is the acceptance ratio and gives the probability of accepting the proposed value.
4. Let $\theta_i^{(t)} = \theta_i^*$with probability min(1, s_t) otherwise let $\theta_i^{(t)} = \theta_i^{(t-1)}$

A popular and more efficient method is the acceptance-rejection (A-R) M-H sampling method which is available. Whenever the target densities are bounded by a density from which it is easy to sample.

2.2 The Gibbs sampler

The Gibbs sampler (Casella and George 1992; Gelfand and Smith 1990; Gilks and Wild 1992) is the special M-H algorithm whereby the proposal density for updating θ_j equals the full conditional $p(\theta_j^*/\theta_j)$ so that proposals are acceptance with probability 1.

The Gibbs sampler involves parameter-by-parameter or block-by-blocks updating, which when completed from the transaction from $\theta^{(t)}$ to $\theta^{(t+1)}$.

$$\begin{aligned}
&1.\ \theta_1^{(t+1)} \approx f_1\left(\theta_1 \middle/ \theta_2^t, \theta_3^{(t)}, \ldots, \theta_D^{(t)}\right)\\
&2.\ \theta_2^{(t+1)} \approx f_2\left(\theta_2 \middle/ \theta_1^{t+1}, \theta_3^{(t)}, \ldots, \theta_D^{(t)}\right)\\
&\vdots\\
&D.\ \theta_D^{(t+1)} \approx f_D\left(\theta_D/\theta_1^{t+1}, \theta_2^{(t+1)}, \ldots, \theta_{D-1}^{(t+1)}\right)
\end{aligned}$$

Repeated sampling from M-H samplers such as the Gibbs samplers generates an autocorrelated sequence of numbers that, subject to regularity condition (ergodicity, etc.), eventually 'forgets' the starting values $\theta^0 = (\theta_1^{(0)},\ \theta_2^{(0)},\ldots,\theta_D^{(0)})$ used to initialize the chain, and converges to a stationary sampling distribution $p(\theta/y)$.

In practice, Gibbs and M-H algorithms are often combined, which results in a "hybrid" MCMC procedure.

3 The stochastic volatility model

3.1 Autoregressive SV model with student's distribution

In this paper, we will consider the p-th order ARSV-t model, ARSV(p)-t, as follows:

$$\begin{cases} Y_t = \sigma\xi \exp(V_t/2) \\ V_t = \phi_1 V_{t-1} + \cdots + \phi_p V_{t-p} + \eta_{t-1} \end{cases}$$

$$\xi_t = \frac{\varepsilon_t}{\sqrt{\kappa_t/(v-2)}}, \quad \kappa_t \approx \chi^2(v)$$

where κ_t is independent of (ε_t, η_t), Y_t is the stock return for indexes market, and V_t is the log-volatility which is assumed to follow a stationarity AR(p) process with a persistent parameter $|\phi| \prec 1$. By this specification, the conditional distribution, ξ_t, follows the standardised t-distribution with mean zero and variance one. Since κ_t is independent of (ε_t, η_t), the correlation coefficient between ξ_t and η_t is also ρ.

If $\phi \approx N(0, 1)$, then;

$$\phi_1 = \frac{\left(\sum_{t=1}^{T} V_t V_{t-1}\right) - \phi_2\left(\sum_{t=1}^{T} V_{t-1}V_{t-2}\right) + \overline{\phi_1}}{\left(\sum_{t=1}^{T} V_{t-1}^2\right) - 1}$$

and

$$\phi_2 = \frac{\left(\sum_{t=2}^{T} V_t V_{t-2}\right) - \phi_1\left(\sum_{t=2}^{T} V_{t-1}V_{t-2}\right) + \overline{\phi_2}}{\left(\sum_{t=2}^{T} V_{t-2}^2\right) - 1}.$$

The conditional posterior distribution of the volatility is given by:

$$p(V/\Theta, Y) \propto e^{\left(\frac{1}{2*\sigma^2}\left(\sum_{t=1}^{T} Y_t^2 e^{-V_t}\right) - \frac{1}{2}\sum_{t=1}^{T}(V_t - \phi_1 V_{t-1} - \phi_2 V_{t-2})^2 - \frac{1}{2}\sum_{t=1}^{T}(V_{t+1} - \phi_1 V_t - \phi_2 V_{t-2})^2\right)}.$$

The representation of the SV-t model in terms of a scale mixture is particularity useful in a MCMC context since it allows for sampling a non-log-concave sampling problem into a log-concave one. This allows for sampling algorithms which guarantee convergence in finite time, (see Frieze et al. 1994) allowing log returns to be student-t-distributed naturally changes the behavior of the stochastic volatility process, in the standard SV model, large value of $|Y_t|$ induce large value of the V_t.

3.2 Basic SVOL model

Jacquier et al. (1994), hereafter JPR, introduced Markov Chain technique (MCMC) for the estimation of the basic SVOL model with normally distributed conditional errors.

$$\begin{cases} Y_t = \sqrt{V_t}\varepsilon_t^s \\ \log(V_t) = \alpha + \delta\log(V_{t-1}) + \sigma_v \varepsilon_t^v \end{cases}$$

$$(\varepsilon_t^s, \varepsilon_t^v) \approx N(0, I_2).$$

Let $\Theta = (\alpha, \delta, \sigma_v)$ be the vector of parameters of the basic SVOL, and $V = (V_t)_{t=1}^T$, where α is the intercept. The parameters vector consists of a location α, a volatility persistence $\delta$and a volatility of volatility σ_v.

The basic SVOL specifies zero correlation, the errors of the mean and variance equations.

Briefly, the Hammersley-Clifford theorem states that having a parameter-set Θ, a state V_t and an observation Y_t, we can obtain the joint distribution $p(\Theta, V/Y)$ from $p(\Theta, V/Y)$and $p(V/\Theta, Y)$, under some mild regularity conditions. Therefore by applying the theorem iteratively, we can break a complicated multidimensional estimation problem into many sample one-dimensional problems.

Creating a Markov Chain $\Theta^{(i)}$ via a Monte-Carlo process, the Ergodic Averaging theorem states that the time-average of a parameter will converge towards its posterior mean.The formula of Bayes factorize the posterior distribution likelihood function with prior hypotheses:

$$P(\Theta, V/Y)\alpha P(Y/V,\Theta)P(V/\Theta)P(\Theta)$$

where α is the intercept, δ the volatility persistence and σ_v is the standard deviation of the shock to log V_t.

We use a Normal-Gamma prior, so, the parameters $\alpha, \delta \approx N$ and $\sigma_v^2 \approx IG$, (Appendix A) Then:

$$P(\alpha, \delta/\sigma_v, V, Y) \approx \prod P(V_t/V_{t-1}, \alpha, \delta, \sigma_v)P(\alpha, \delta)\alpha N$$

And for σ_v, we obtain:

$$P(\sigma^2/\alpha, \sigma_v, V, Y)\alpha \prod P(V_t/V_{t-1}, \alpha, \delta, \sigma_v)P(\sigma_v^2)\alpha IG$$

4 Empirical illustration

4.1 The data

Our empirical analysis focuses on the study of five international financial indexes: the Dow Jones Industrial, the Nikkei, the CAC40, the S&P500 and the Nasdaq. The indices are compiled and provided by Morgan Stanley Capital International. The returns are defined as $y_t = 100 \times (\log S_t - \log S_{t-1})$. We used the last 2,252 observations for all indices except the Nikkei, when we have only used 2,201 observations due to flack of data. The daily stock market indices are for five different countries over the period 01/01/2000–31/12/2008.

Table 1 reports the mean, standard deviation, median, and the empirical skewness as well as kurtosis of the five series. All series reveal negative skewness and overkurtosis which is a common finding of financial returns.

4.2 Estimation of SV models

The standard SV model is estimated by running the Gibbs and A-R M-H algorithm based on 15,000 MCMC iterations, where 5,000 iterations are used as burn-in period.

Tables 2 and 3 show the estimation results in the basic SVOL model and the SV-t model of the daily indexes. α and δ are independent priors. The prior in δ is essentially flat over

Table 1 Summary statistics for daily returns

	Mean	SD	Median	Skewness	Kurtosis
CAC 40	3.7E-04	0.013	5.0e-4	−0.295	5.455
Dow Jones	2.8e-04	0.015	4.0e-4	−0.368	4.522
Nasdaq	2.5e-04	0.014	5.5e-4	−0.523	6.237
Nikkei	3.5e-04	0.005	3.2e-4	−0.698	3.268
S&P	2.8e-04	0.008	4.5e-4	−0.523	5.659

Table 2 Estimation results for the SVOL model

	CAC40	DOWNJONES	NASDAQ	NIKKEI	S&P
σ	0.4317 (0.0312)	0.4561 (0.0421)	0.5103 (0.0393)	0.5386 (0.0523)	0.4435 (0.0623)
α	−0.1270 (0.0421)	0.0059 (0.0534)	0.1596 (0.0332)	0.1966 (0.0493)	−0.1285 (0.0593)
δ	−0.7821 (0.0621)	0.0673 (0.0317)	0.6112 (0.0429)	0.8535 (0.0645)	0.7224 (0.0423)

[0,1]. We impose stationarity for log (V_t) by truncating the prior of δ. Other priors for δ are possible.

Geweke (1994a, b) propose alternative priors to allow the formulation of odds ratios for non stationarity. Whereas Kim et al. (1998) center an informative Beta Prior around 0.9.

Table 2 shows the results for the daily indexes. The posterior of δ are higher for the daily series. The highest mean is 0.782, 0.067, 0.611, 0.85 and 0.722, for the full sample NIKKEI.

This result is not a priori curious because the model of Jacquier et al. (1994) can lead to biased volatility forecast.

Well, as the basic SVOL, there is no apparent evidence of unit of volatility. There are other factors that can deflect this rate such exchange rate (O'Brien and Dolde 2000).

We deduce from this model, against the empirical evidence, positive and negative shocks have the same effect in volatility.

Table 3 shows the Metropolis Hasting estimates of the Autoregressive SV model.

The estimates of are between 0.554 and 0.643, while those of σ are between 0.15 and 0.205.

Against, the posterior of for the SV-t model are located higher.[1] This is consistent with temporal aggregation (as suggested by Meddahi and Renault (2000)). This result confirms the typical persistence reported in the GARCH literature. After result, the first volatility factors have higher persistence while the small values of Φ_2 indicate the low persistence of the second volatility factors.

The second factor Φ_2 plays an important role in the sense that it captures extreme values, which may produce the leverage effect, then it can considered conceivable.

The estimates of ρ are negative in most cases. Another thing to note is that these estimates are relatively higher than that observed by Asai et al. (2006) and Manabu Asai (2008). The estimated of ρ for index S&P using Monte-Carlo simulation is −0.3117, then it is −0.0235 using Metropolis Hasting. This implies that for each data set, the innovations in the mean and volatility are negatively correlated.

[1] We choose $p = 2$ because if $p = 1$ and $v \to \infty$, the ARSV-t model declined to the asymmetric SV model of Harvey and Shephard (1996).

Table 3 Estimation results for the SV-t model

CAC40	DOWNJONES	NASDAQ	NIKKEI	S&P	
Φ_1	0.4548 (0.0037)	0.40839 (0.0021)	0.5225 (0.0065)	0.4348 (0.0059)	0.2890 (0.0046)
Φ_2	0.5544 (0.1524)	0.6437 (0.1789)	0.4473 (0.1326)	0.4865 (0.1628)	0.6133 (0.1856)
σ	0.0154 (0.0294)	0.0205 (0.0367)	0.0131 (0.0524)	0.0148 (0.0689)	0.0135 (0.0312)
ρ	−0.02191 (0.0625)	−0.0306 (0.0346)	−0.0489 (0.0498)	0.0751 (0.0255)	−0.0235 (0.0568)

Negative correlations between mean and variance errors can produce a “Leverage” effect in which negative (positive) shocks to the mean are associated with increases (decreases)in volatility.

The return of different indexes is not only affected by market structure (Sharma 2010) but also is deeply influenced by different crises observed in international market, i.e., the Asian crises detected in 1987 and the Russian one in 2002. The markets in our sample are subject to several crises that directly affect the evolution of the return indexes. The event of 11 September 2002, the Russian crisis and especially the beginning of the subprime crisis in the United States in July 2007 justify our results. Theses results explored in Fig. 1 suggest that periods of market crisis or stress increases the volatility. Then the volatility at time (t) depends on the volatility at ($t-1$) (Engle 1982).

When the new information coming in the market, it can be disrupted and this affects the anticipation of shareholders for the evolution of the return.

The resulting plots of the smoothed volatilities are shown in Fig. 2. We take our analysis in the Nikkei indices, but the other are reported in “Appendix B”.

The convergence is very remarkable for the Nikkei, like Down Jones, Nasdaq and the CAC40 indexes. This enhances the idea that the algorithm used for estimated volatility is a good choice.

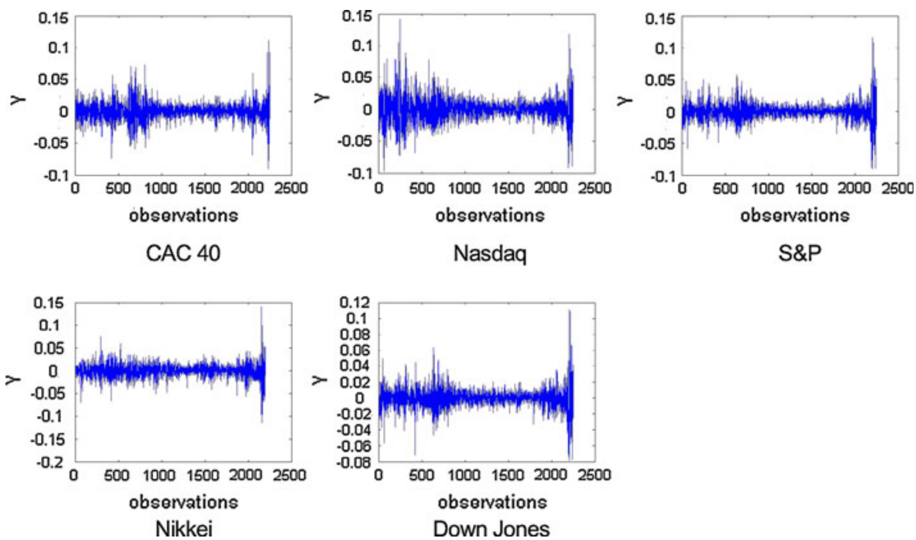

Fig. 1 Return for indexes

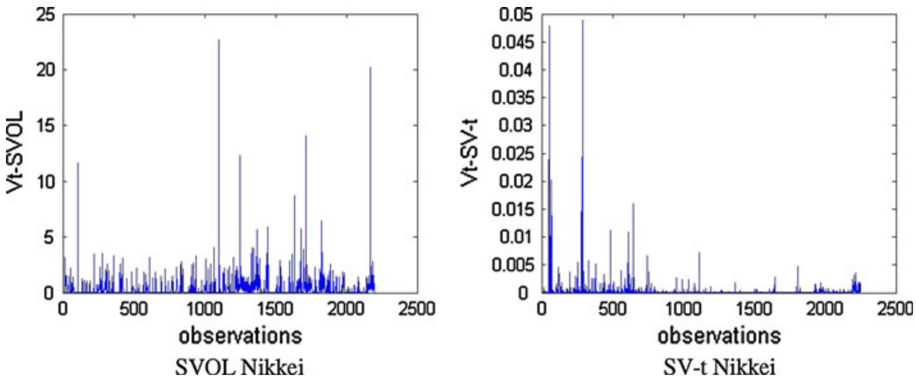

Fig. 2 Smoothed estimates of Vt, Basic SVOL and SV-t model

The basic SVOL model mis-specified can induce substantial parameter bias and error in inference about V_t, Geweke (1994a, b) showed that the basic SVOL has the same problem with the largest outlier, October 1987 "Asiatique crisis". The V_t for the model SVOL reveal a big outlier on period crises.

The corresponding plots of innovation are given by Fig. 3 for two model basic SVOL and SV-t model for Nikkei indexes. Appendix C shows the QQplot for the other indices respectively for the Nasdaq, S&P, the Down Jones and the CAC40 for the two models. The standardized innovation reveals a big outlier when the market in stress (Hwang and Salmon 2004).

The advantages of Asymetric basic SV is able to capture some aspects of financial market and the main properties of their volatility behavior (Danielsson 1994; Eraker et al. 2000).

It is shown that the inclusion of student-t errors improves the distributional properties of the model only slightly. Actually, we observe that basic SVOL model is not able to capture extreme observation in the tail of the distribution. In contrast, the SV-t model turns out to be more appropriate to accommodate outliers. The corresponding plot of innovation for the basic model is unable to capture the distribution properties of the returns. This is confirmed

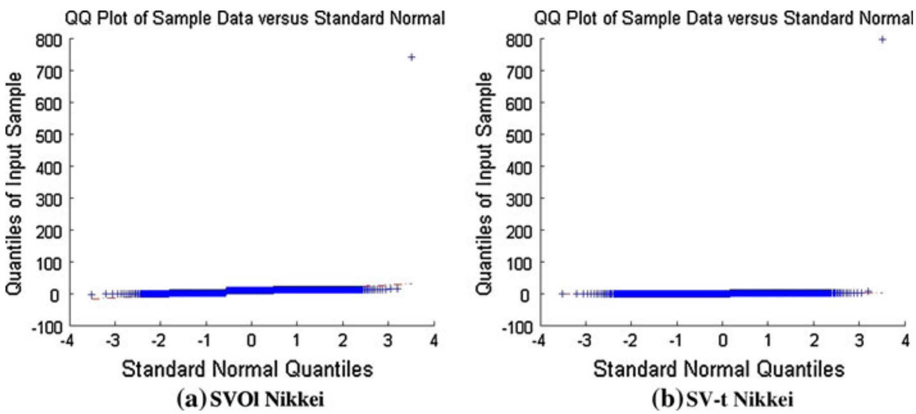

Fig. 3 QQ plot of normalized innovation based on the basic SVOL model (*left*) and the SV-t model (*right*)

Table 4 MSE and likelihood for two models

	CAC 40		Down Jones		Nasdaq	
	SVOL	SV-t	SVOL	SV-t	SVOL	SV-t
MSE	0.0210	0.0296	0.1277	0.0040	0.2035	0.0229
Likelihood	$-2.595.10^{-4}$	−0.0046	−0.0374	−0.0012	−0.2257	−0.0158

	Nikkei		S&P	
	SVOL	SV-t	SVOL	SV-t
MSE	0.0241	0.0168	0.0248	0.0395
Likelihood	$-0.2257.10^{-4}$	−0.0094	−0.0054	0.02016

by the Jarque–Bera normality test and the QQ plot revealing departures from normality, mainly stemming from extreme innovation.

Finally, in order to detect which of the two models is better, we opt for two indicator of performance, such as the likelihood and the MSE. Likelihood is a function of the parameters of the statistical model that plays a preponderant role in statistical inference. MSE is called Squared error loss and it measures the average of the square of "error". Table 4 reveals the results for this measure, and indicates that the

SV-t model is much more efficient than the other. Indeed, in term of comparison, we interested in the convergence of two models. We find that convergence to the SV-t model is fast.

Table 4 shows the performance of the algorithm and the consequence of using the wrong model on the estimates of volatility. The efficiency is at 60%.

The MCMC is more efficient for all parameters used in this two model. A certain threshold, all parameters are stable and converge to a certain level. Appendix D and E show that the α, δ, σ, converge and stabilize, this shows the power for MCMC.

The results for both simulated show that the algorithm of SV-t model is fast and converges rapidly with acceptable levels of numerical efficiency. Then, our sampling provides strong evidence of convergence of the chain.

5 Conclusion

We have applied these MCMC methods to the study of various indexes. The ARSV-t models were compared with SVOL models of Jacquier et al. (1994) models using S&P, the Down Jones, Nasdaq, Nikkei and the CAC40. The empirical results show that SV-t model can describe extreme values to a certain extent, but it is more appropriate to accommodate outliers. Surprisingly, we have frequently observed that the best model is the Student's t-distribution (ARSV-t) with their forecast performance. Our result confirms the finding from Manabu Asai (2008), who indicates first, that the ARSV-t model provides a better fit than the MFSV model and second, the positive and negative shocks haven't the same effect in volatility. Our result proves the efficiency of Markov Chain for our sample and the convergence and stability for all parameter to a certain level. This paper has made certain contributions, but several extensions are still possible. To find the best results, opt for extensions of SVOL.

Appendix A

The posterior volatility is

$$P(V/\Theta, V) \propto P(Y/\Theta, V)P(V/\Theta) \propto \prod_{t=1}^{T} P(V_t/V_{t-1}, V_{t+1}, \Theta, Y_t)Z$$

with

$$P(V/V_{t-1}, V_{t+1}, \Theta, Y_t) \propto P(Y_t/V_t, \Theta)P(V_t/V_{t-1}, \Theta)P(V_{t+1}/V_t, \Theta).Z$$

A simple calculation shows that:

$$\prod(V_t) = P(V_t/V_{t-1}, V_{t+1}, \Theta, Y_t) \propto \frac{1}{V_t^{0.5}} \exp\left(\frac{-Y_t^2}{2V_t}\right) \frac{1}{V_t} \exp\left(-\frac{(\log V_t - \mu_t)^2}{2\sigma^2}\right)$$

with

$$\mu_t = \frac{\alpha(1-\beta) + \beta(\log V_{t+1} + \log V_{t-1})}{1+\beta^2}$$

and

$$\sigma^2 = \frac{\sigma_v^2}{1+\beta^2}.$$

The MCMC algorithm consists of the following steps:

$$P(\alpha, \delta/\sigma_v, V, Y) \approx N$$

$$P\left(\sigma_v^{2/}\alpha, \delta, V, Y\right) \approx IG$$

$$P(V_t/V_{t-1}, V_{t+1}, \Theta, Y_t) : Metropolis - Hasting.$$

A iteration (j),

$$\alpha^{(j)} = \frac{\sum_{t=1}^{T} \log V_t^{(j-1)} - \beta^{(j-1)} \sum_{t=1}^{T} \log V_{t-1}^{(j-1)}}{(\sigma_v^2)^{(j-1)} + T}.$$

By following the same approach, the estimator δ at step (j) is given by:

$$\delta^{(j)} = \frac{\sum_{t=1}^{T} \left[\log V_{t-1}^{(j-1)} \left(\log V_t^{(j-1)} - \alpha^{(j)}\right)\right]}{(\sigma_v^2)^{(j-1)} + \sum_{t=1}^{T} \left(\log V_{t-1}^{(j-1)}\right)^2}.$$

For parameter σ_v^2, the prior density is an inverse gamma (IG (a, b)). The expression of the estimator parameter σ_v^2 at step (j) is given by:

$$(\sigma_v^2)^{(j)} = \frac{\frac{1}{2}\sum_{t=1}^{T} (\log V_t^{(j-1)} - \alpha^{(j)} - \delta^{(j)} \log V_{t-1}^{(j-1)})^2 + b}{T/2 + a - 1}.$$

Appendix B

See Fig. 4.

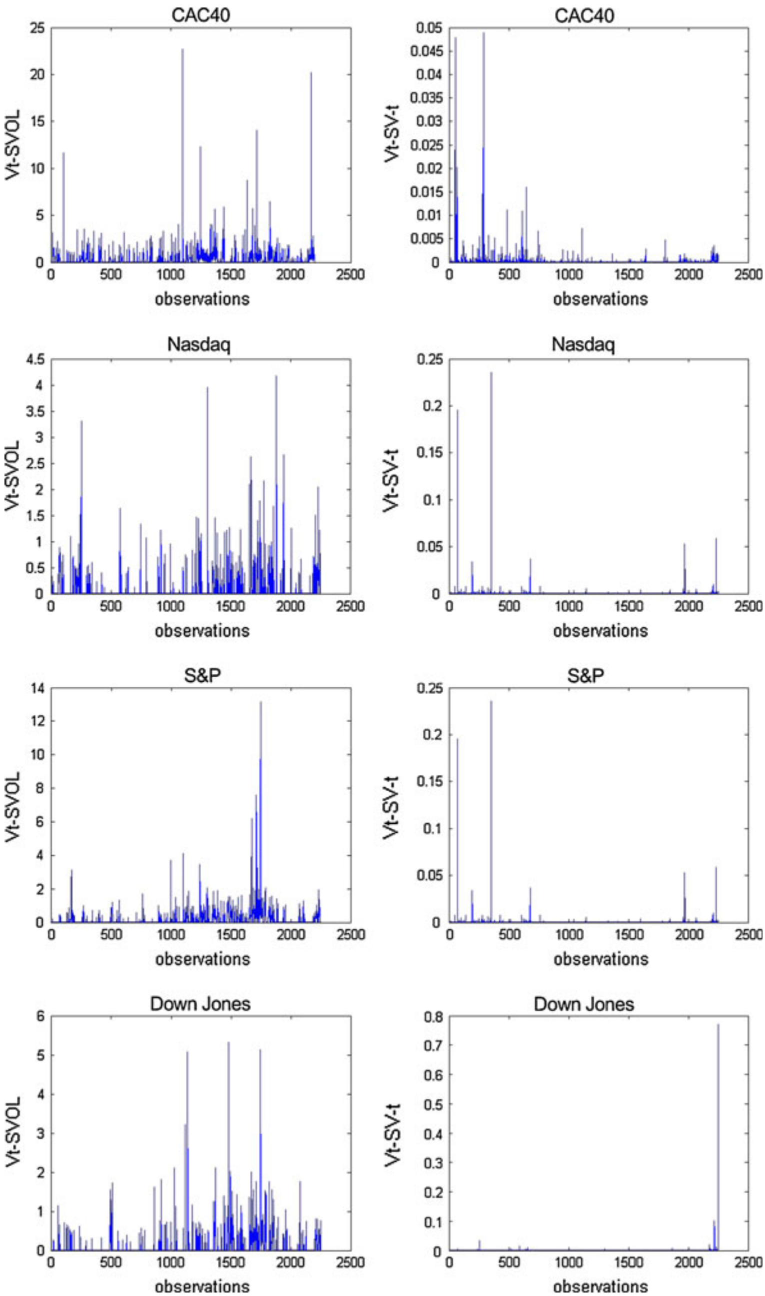

Fig. 4 Smoothed estimates of Vt, Basic SVOL and SV-t model

Appendix C

See Fig. 5.

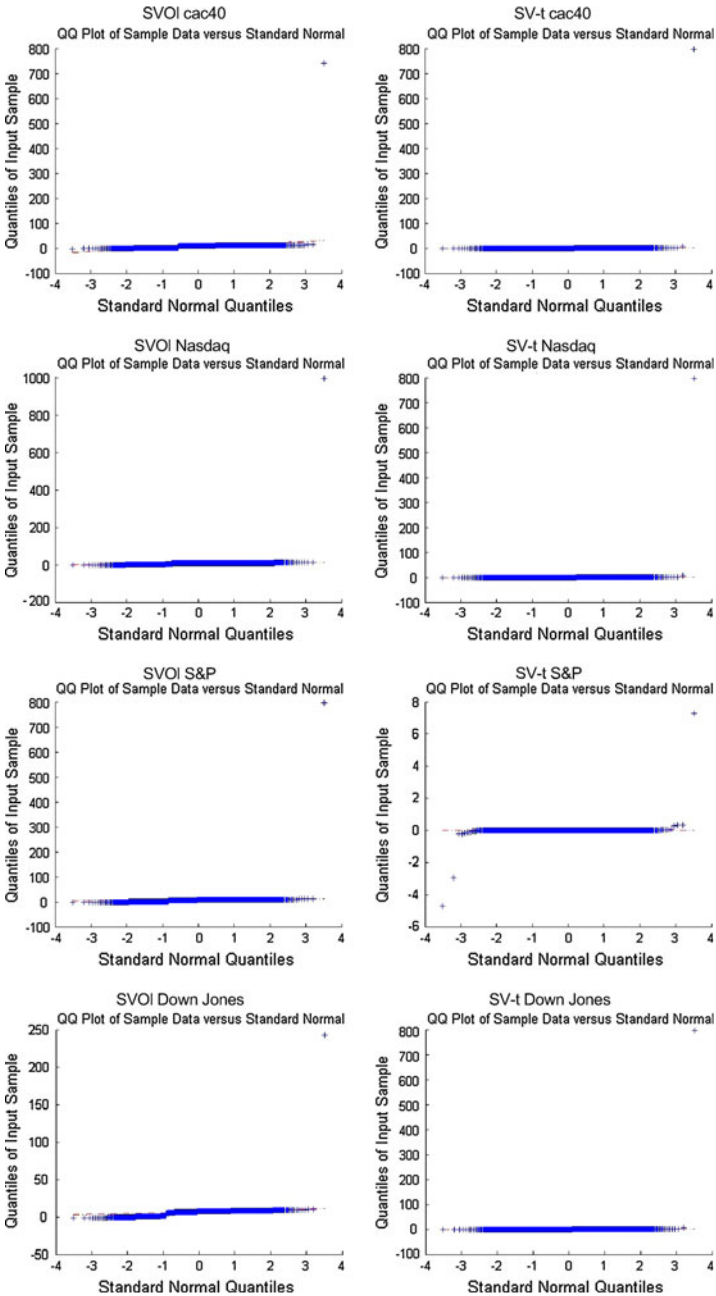

Fig. 5 Smoothed estimates of Vt, Basic SVOL and SV-t model

Appendix D

See Fig. 6.

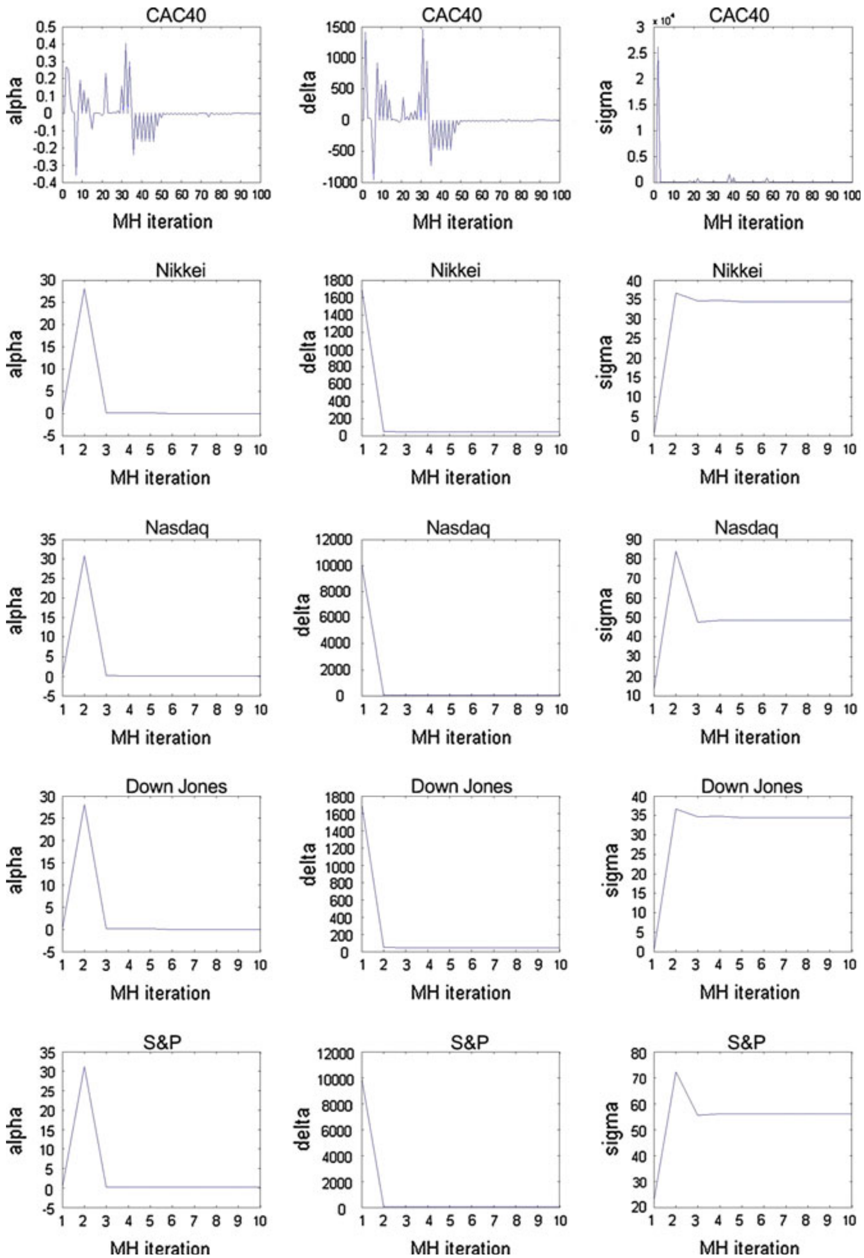

Fig. 6 Behavioral of parameters of Basic SVOL model

Appendix E

See Fig. 7.

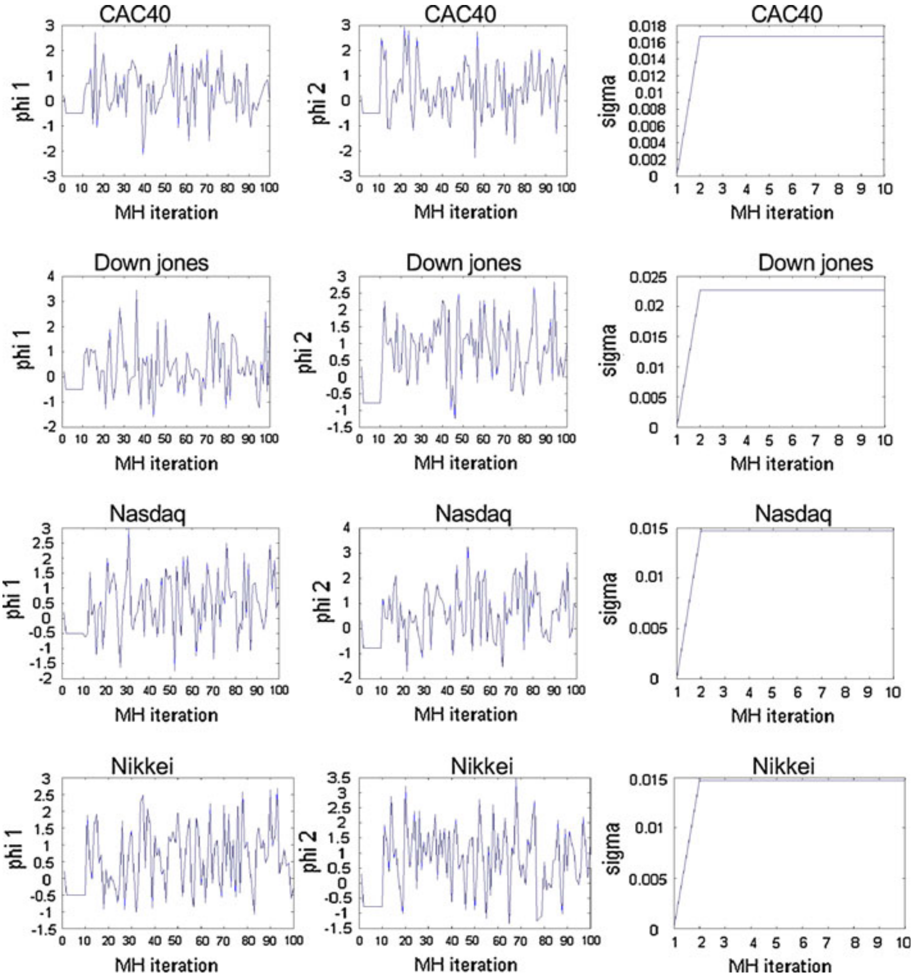

Fig. 7 Behavioral of parameters of SV-t model

References

Asai M (2008) Autoregressive stochastic volatility models with heavy-tailed distributions: a comparison with multifactors volatility models. J Empir Finance 15:322–345

Asai M, McAleer M, Yu J (2006) Multivariate stochastic volatility. Econom Rev 25(2–3):145–175

Casella G, George E (1992) Explaining the Gibbs sampler. Am Stat 46(3):167–174

Chang R, Huang M, Lee F, Lu M (2007) The jump behavior of foreign exchange market: analysis of Thai Baht. Rev Pac Basin Financial Mark Policies 10(2):265–288

Chib S, Nardari F, Shephard N (2002) Markov Chain Monte Carlo methods for stochastic volatility models. J Econom 108:281–316

Danielsson J (1994) Stochastic volatility in asset prices: estimation with simulated maximum likelihood. J Econom 61:375–400
Engle R (1982) Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. Econometrica 50:987–1007
Eraker B, Johannes M, Polson N (2000) The impact of jumps in volatility and return. Working paper, University of Chicago
Frieze AM, Kannan R, Polson N (1994) Sampling from log-concave distributions. Ann Appl Probab 4:812–837
Gelfand AE, Smith AFM (1990) Sampling-based approaches to calculating marginal densities. J Am Stat Assoc 85:398–409
Geweke J (1994a) Priors for macroeconomic time series and their application. Econom Theory 10:609–632
Geweke J (1994b) Comment on Bayesian analysis of stochastic volatility. J Bus Econ Stat 12(4):371–417
Gilks WR, Wild P (1992) Adaptive rejection sampling for Gibbs sampling. J Roy Stat Soc Ser C 41:337–348
Harvey AC, Shephard N (1996) Estimation of an asymmetric stochastic volatility model for asset returns. J Bus Econom Stat 14:429–434
Hastings WK (1970) Monte Carlo sampling methods using Markov chains and their applications. Biometrika 57:97–109
Hsu D, Chiao CH (2010) Relative accuracy of analysts' earnings forecasts over time: a Markov chain analysis. Rev Quant Finance Account, (forthcoming)
Hwang S, Salmon M (2004) Market stress and herding. J Empir Finance 11:585–616
Jacquier E, Polson N, Rossi P (1994) Bayesian analysis of stochastic volatility models (with discussion). J Bus Econ Stat 12(4):371–417
Kim SN, Shephard N, Chib S (1998) Stochastic volatility: likelihood inference and comparison with ARCH models. Rev Econom Stud 65:361–393
Meddahi N, Renault E (2000) Temporal aggregation of volatility models. Document de Travail CIRANO, 2000–2022
O'Brien TJ, Dolde W (2000) A currency index global capital asset pricing model, with Thomas J. O'Brien. Eur Finan Manage 6(1):7–18
Pitt Shephard (1997) Likelihood analysis of Non-Gaussian measurement time series. Biometrika 84(3):653–667
Sharma V (2010) Stock return and product market competition: beyond industry concentration. Rev Quant Finance Account. doi:10.1007/s11156-010-0205-0